Methodisch geordnete Aufgabensammlung

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XXVIII. Quadratische Gleichungen mit drei und vier Unbekannten

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46. $$\left| \begin{array}{l} x^4 - y^4 = a(x^3 - y^3) \\ x^3 + y^3 = b(x^2 + y^2) \end{array} \right|$$

47. $$\left| \begin{array}{l} x^3 + y^3 = a(x^2 - y^2) \\ x^4 + y^4 = b(x^3 - y^3) \end{array} \right|$$

48. $$\left| \begin{array}{c} x^4 + x^2y^2 + y^4 = 2a(x + y)^2 \\ x^5 + y^5 = 2b(x + y)^3 \end{array} \right|$$

49. $$\left| \begin{array}{c} x(x + y)(x + 2y)(x + 3y) = 840 \\ (x + \quad y)^2 - (x + y)^2 = 11 \end{array} \right|$$

50. $$\left| \begin{array}{l} \frac{1 + xy}{x + y} + \frac{x + y}{1 + xy} = 2a \\ \frac{1 - xy}{x - y} + \frac{x - y}{1 - xy} = 2b \end{array} \right|$$

51. $$\left| \begin{array}{l} \frac{1 - xy}{x + y} + \frac{x + y}{1 - xy} = \frac{2}{a} \\ \frac{1 + xy}{x - y} + \frac{x - y}{1 + xy} = \frac{2}{b} \end{array} \right|$$

XXVIII.

Quadratische Gleichungen mit drei und vier Unbekannten.

A. Mit drei Unbekannten.

1. $$\left| \begin{array}{l} x : y : z = a : b : c \\ x^2 + y^2 + z^2 = m^2 \end{array} \right|$$

2. $$\left| \begin{array}{l} x(x + y + z) = a \\ y(x + y + z) = b \\ z(x + y + z) = c \end{array} \right|$$

3. $$\left| \begin{array}{l} (y + z)(x + y + z) = a \\ (x + z)(x + y + z) = b \\ (x + y)(x + y + z) = c \end{array} \right|$$

4. $$\left| \begin{array}{l} (y + z) : (x + z) : (x + y) = a : b : c \\ (y + z)^2 + (x + z)^2 + (x + y)^2 = 1 \end{array} \right|$$

5. $$\left| \begin{array}{c} 2x - 4y + z = 0 \\ x + y - 4z = 0 \\ (x + 1)(z + 1) = (y - 1)(y + 6) \end{array} \right|$$

6. $\left| \begin{aligned} ax &= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \\ by &= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \\ cz &= \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \end{aligned} \right|$

7. $\left| \begin{aligned} \frac{xyz}{y+z} &= a \\ \frac{xyz}{x+z} &= b \\ \frac{xyz}{x+y} &= c \end{aligned} \right|$

8. $\left| \begin{aligned} \frac{x^3+y^3+z^3}{y+z-x} &= a \\ \frac{x^3+y^3+z^3}{x+z-y} &= b \\ \frac{x^3+y^3+z^3}{x+y-z} &= c \end{aligned} \right|$

9. $\left| \begin{aligned} x^2yz &= a \\ xy^2z &= b \\ xyz^2 &= c \end{aligned} \right|$

10. $\left| \begin{aligned} yz &= a \\ xz &= b \\ xy &= c \end{aligned} \right.$

11. $\left| \begin{aligned} x &= ayz \\ y &= bxz \\ z &= cxy \end{aligned} \right|$

12. $\left| \begin{aligned} x^3 &= ayz \\ y^3 &= bxz \\ z^3 &= cxy \end{aligned} \right|$

13. $\left| \begin{aligned} x(y+z) &= a \\ y(x+z) &= b \\ z(x+y) &= c \end{aligned} \right|$

14. $\left| \begin{aligned} (x+y)(x+z) &= a \\ (x+y)(y+z) &= b \\ (x+z)(y+z) &= c \end{aligned} \right|$

15. $\left| \begin{aligned} x(x+y+z) &= a - yz \\ y(x+y+z) &= b - xz \\ z(x+y+z) &= c - xy \end{aligned} \right|$

16. $\left| \begin{aligned} (x+y-z)(x-y+z) &= a \\ (y+z-x)(y-z+x) &= b \\ (z+x-y)(z-x+y) &= c \end{aligned} \right|$

17. $\left| \begin{aligned} x^2 - (y-z)^2 &= a \\ y^2 - (x-z)^2 &= b \\ z^2 - (x-y)^2 &= c \end{aligned} \right|$

18. $\left| \begin{aligned} (y+z)(2x+y+z) &= b+c \\ (x+z)(2y+x+z) &= a+c \\ (x+y)(2z+x+y) &= a+b \end{aligned} \right|$

19. $$\left| \begin{array}{l} x = a^2 (x + y + z) yz \\ y = b^2 (x + y + z) xz \\ z = c^2 (x + y + z) xy \end{array} \right|$$

20. $$\left| \begin{array}{l} \frac{y}{z} + \frac{z}{y} = \frac{a}{x} \\ \frac{x}{z} + \frac{z}{x} = \frac{b}{y} \\ \frac{x}{y} + \frac{y}{x} = \frac{c}{z} \end{array} \right|$$

21. $$\left| \begin{array}{l} x \left(1 - \frac{x}{a^2}\right) + y + z = 0 \\ y \left(1 - \frac{y}{b^2}\right) + x + z = 0 \\ z \left(1 - \frac{z}{c^2}\right) + x + y = 0 \end{array} \right|$$

22. $$\left| \begin{array}{l} \frac{yz}{y + z} = \frac{xyz}{b + c} - x \\ \frac{xz}{x + z} = \frac{xyz}{a + c} - y \\ \frac{xy}{x + y} = \frac{xyz}{a + b} - z \end{array} \right|$$

23. $$\left| \begin{array}{l} x + y = au \\ x - y = bu \\ x^2 + y^2 = cu \end{array} \right|$$

24. $$\left| \begin{array}{l} x + y = 5u \\ x - y = 2u \\ x^3 + y^3 = 185u \end{array} \right|$$

25. $$\left| \begin{array}{l} x + y = 2u \\ x^2 + y^2 = 5u \\ x^3 + y^3 = 7u^2 \end{array} \right|$$

26. $$\left| \begin{array}{l} x + y = au \\ x^2 + y^2 = bu^2 \\ x^5 + y^5 = c^4 u \end{array} \right|$$

27. $$\left| \begin{array}{l} x + y = au \\ x^2 + y^2 = \frac{b}{u} \\ x^3 + y^3 = c \end{array} \right|$$

28. $$\left| \begin{array}{l} \frac{1}{x^2} + \frac{1}{y^2} = \frac{2a}{u^2} \\ \frac{1}{x^2} - \frac{1}{y^2} = \frac{2b}{u^2} \\ \frac{1}{x} + \frac{1}{y} = \frac{1}{c} \end{array} \right|$$

29. $$\left| \begin{array}{l} x : y = y : z \\ x + y + z = 19 \\ x^2 + y^2 + z^2 = 133 \end{array} \right|$$

30. $$\left| \begin{array}{l} x : y = y : z \\ x + y + z = 21 \\ (x - y)^2 + (x - z)^2 + (y - z)^2 = 126 \end{array} \right|$$

31. $$\left| \begin{array}{l} x + y = 2az \\ x^2 + y^2 = 2bz^2 \\ x^n + y^n + z^n = c^n \end{array} \right|$$

32. $$\left| \begin{array}{l} (1 - xy)(z + 1) = 2 \\ (x - y)(z + 1) = 2a \\ (x^2 - y^2)(z + 1)^2 = 4bz \end{array} \right|$$

33. $\left| \begin{array}{l} x(y-1)(u-1) = 2a \\ x^2(y^2-1)(u-1)^2 = 4bu \\ x^3(y^3-1)(u-1)^3 = 6cu^2 \end{array} \right|$

34. $\left| \begin{array}{l} \frac{x(u-1)}{y-1} = a \\ \frac{x^2(u^2-1)}{y^2-1} = b \\ \frac{x^3(u^3-1)}{y^3-1} = c \end{array} \right|$

35. $\left| \begin{array}{l} \frac{x(u-1)}{y-1} = a \\ \frac{x^2(u^2-1)}{y^2-1} = b \\ \frac{x^4(u^4-1)}{y^4-1} = c \end{array} \right|$

36. $\left| \begin{array}{l} x^2 + (y-z)^2 = a \\ y^2 + (x-z)^2 = b \\ z^2 + (x-y)^2 = c \end{array} \right|$

37. $\left| \begin{array}{l} x^2 - yz = a \\ y^2 - xz = b \\ z^2 - xy = c \end{array} \right|$

38. $\left| \begin{array}{l} y^2 + z^2 - x(y+z) = a \\ x^2 + z^2 - y(x+z) = b \\ x^2 + y^2 - z(x+y) = c \end{array} \right|$

39. $\left| \begin{array}{l} 2x^2 + y^2 - yz + z^2 = 9 \\ 2y^2 + x^2 - xz + z^2 = 6 \\ 2z^2 + x^2 - xy + y^2 = 3 \end{array} \right|$

40. $\left| \begin{array}{l} 2x^2 + x(y+z) - yz = a \\ 2y^2 + y(x+z) - xz = b \\ 2z^2 + z(x+y) - xy = c \end{array} \right|$

41. $\left| \begin{array}{l} y^2 + yz + z^2 = a^2 \\ x^2 + xz + z^2 = b^2 \\ x^2 + xy + y^2 = c^2 \end{array} \right|$

42. $\left| \begin{array}{l} \frac{x^3 - y^3}{(x+y)^3} = \frac{a}{u} \\ \frac{x^3 + y^3}{(x-y)^3} = \frac{u}{b} \\ \frac{(x-y)^2}{x+y} = \frac{c}{u} \end{array} \right|$

43. $\left|\begin{array}{l} cx + (b - a)\,z = yz \\ ay + (c - b)\,x = xz \\ bz + (a - c)\,y = xy \end{array}\right|$ *) 44. $\left|\begin{array}{l} ax + bz - cy = x^2 \\ by + cx - az = y^2 \\ cz + ay - bx = z^2 \end{array}\right|$

45. $\left|\begin{array}{l} (a + b)(x + y) - (b + c)\,z = 2yz \\ (b + c)(y + z) - (a + c)\,x = 2xz \\ (a + c)(x + z) - (a + b)\,y = 2xy \end{array}\right|$

46. $\left|\begin{array}{l} (a + b)(x - y) + 2\,(a - b)\,z = x^2 - y^2 \\ (b + c)(y - z) + 2\,(b - c)\,x = y^2 - z^2 \\ (c + a)(z - x) + 2\,(c - a)\,y = z^2 - x^2 \end{array}\right|$

B. Mit vier Unbekannten.

47. $\left|\begin{array}{l} x + y = 7 \\ u + v = 3 \\ x + u^2 = 8 \\ y + v^2 = 4 \end{array}\right|$ 48. $\left|\begin{array}{l} x + y = 12 \\ u + v = 4 \\ x^2 + u^2 = 34 \\ y^2 + v^2 = 50 \end{array}\right|$

49. $\left|\begin{array}{l} xy = uv \\ x + y = 16 \\ u + v = 14 \\ \frac{x}{v} + \frac{u}{y} = 4 \end{array}\right|$ 50. $\left|\begin{array}{l} xy = uv \\ x + y = a \\ u + v = b \\ \frac{x + u}{y + v} = c \end{array}\right|$

51. $\left|\begin{array}{l} xy = a \\ uv = a \\ x + u = b \\ y + v = c \end{array}\right|$ 52. $\left|\begin{array}{l} xy = 24 \\ uv = 6 \\ x + u = 14 \\ y + v = 4 \end{array}\right|$

53. $\left|\begin{array}{l} xy = uv \\ x^2 + y^2 = a \\ u^2 + v^2 = b \\ x + y + u + v = c \end{array}\right|$ 54. $\left|\begin{array}{l} x^2 + y^2 = a \\ u^2 + v^2 = b \\ xy + uv = c \\ xyuv = d \end{array}\right|$

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*) Die Gleichungen 43.—46. sind ihrer Eigenthümlichkeit wegen hier angegeben. Ihre direkte Auflösung führt auf eine Gleichung des vierten Grades. Man findet aber eine Lösung, wenn man nach a, b und c auflöst.

55. $\left|\begin{array}{l} x + y = 16 \\ u + v = 12 \\ xy + uv = 95 \\ xu + yv = 100 \end{array}\right|$

56. $\left|\begin{array}{l} x^2 + y^2 = 17 \\ u^2 + v^2 = 13 \\ xy + uv = 10 \\ xu + yv = 14 \end{array}\right|$

57. $\left|\begin{array}{l} x^2 + y^2 = a \\ u^2 + v^2 = b \\ ux + vy = c \\ vx + uy = d \end{array}\right|$

58. $\left|\begin{array}{l} 2x = y\,(1 + x^2) \\ 2y = u\,(1 + y^2) \\ 2u = v\,(1 + u^2) \\ 2v = a\,(1 + v^2) \end{array}\right|$

59. $\left|\begin{array}{l} x + y = u \\ x + u = v \\ \frac{x - y}{u - v} = a \\ x^2 + y^2 + u^2 + v^2 = 3m^2 \end{array}\right|$

60. $\left|\begin{array}{l} x + y = v \\ x + u = y \\ \frac{3x - v}{3y - u} = a \\ x^2 + y^2 + u^2 + v^2 = 15m^2 \end{array}\right|$

61. $\left|\begin{array}{l} (x + y)^2 + (u + v)^2 = a \\ (x + u)^2 + (y + v)^2 = b \\ (x + v)^2 + (y + u)^2 = c \\ x + y + u + v = m \end{array}\right|$

62. $\left|\begin{array}{l} \frac{x}{y + z} = \frac{u - a}{a},\ \frac{y}{x + z} = \frac{u - b}{b},\ \frac{z}{x + y} = \frac{u - c}{c}, \\ (a + b + c)^2\,(x^2 + y^2 + z^2) + 4(x + y + z)^2\,u^2 = m^2 \end{array}\right|$

63. $\left|\begin{array}{c} \frac{x}{y + z} = \frac{2a - u}{a - 2u},\ \frac{y}{x + z} = \frac{2b - u}{b - 2u},\ \frac{z}{x + y} = \frac{2c - u}{c - 2u} \\ x^2 + y^2 + z^2 = m^2 \end{array}\right|$

XXIX.

Anwendung der quadratischen Gleichungen mit mehreren Unbekannten.

1. Zwei Zahlen verhalten sich wie 5 : 3; ihr Produkt ist 735. Wie heißen dieselben?

2. Welche zwei Zahlen verhalten sich wie 3 : 13, während das Quadrat der ersten um 378 kleiner ist, als das 9fache der zweiten Zahl?

3. Die Zahl 100 in zwei Theile zu zerlegen, daß die Summe der Quadrate derselben 5882 beträgt.

4. Die Zahl 84 in zwei Theile zu zerlegen, daß ihr Produkt 1728 beträgt.