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XXVII. Quadratische Gleichungen mit zwei Unbekannten

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findet ein Maximum statt; sind sie größer, ein Minimum; ist der eine kleiner und der andere größer, keins von beiden. — Bei den Ausdrücken 145—148 findet meist nur ein Maximum oder ein Minimum statt; sonst muß man die Doppeldeutigkeit der Quadratwurzel in Betracht ziehen.

Untersuche nach einer der angegebenen Methoden, ob folgende Ausdrücke ein Maximum oder ein Minimum haben, für welchen Werth von x ein solches eintritt und wie groß dasselbe ist.

134. $x^2 + \frac{16}{x^2}$ $\frac{x^2 - 9}{2x}$ $3x^2 + \frac{2}{x^3}$ $\frac{3x}{2} + \frac{8}{x^3}$
135. $\frac{x^2 + 3}{x + 1}$ $\frac{x^2 - 5}{x - 3}$ $\frac{x - 4}{x^2 - 7}$ $\frac{x - 2}{x^2 + 5}$
136. $\frac{x^2 + 6x + 9}{3x + 4}$, $\frac{x^2 - 5x + 9}{x - 5}$, $\frac{2x + 3}{x^2 + 2x + 7}$, $\frac{2x - 5}{x^2 - 6x + 11}$
137. $\frac{x^2 - 2x + 1}{x^2 - 4x + 5}$, $\frac{3x^2 + 6x - 1}{x^2 - 2x + 2}$, $\frac{7x^2 + 2x + 3}{9x^2 + 6x - 2}$, $\frac{19 + 2x - x^2}{15 + 6x + x^2}$
138. $\frac{9}{x - 3} - \frac{1}{x - 5}$ $\frac{9}{x - 1} - \frac{4}{x - 6}$
139. $\frac{4}{x - 3} - \frac{16}{x - 7}$ $\frac{25}{7 - x} - \frac{9}{3 - x}$
140. $x^3 - x^2 - 16x + 10$ $x^3 - 13x^2 - 64x + 32$
141. $x^3 - 11x^2 - 16x + 98$, $x^3 - 12x^2 + 45x - 10$
142. $x\sqrt{2x - 3}$, $x\sqrt{2x - 3}$ $x\sqrt{x^2 - 4}$, $x\sqrt{x^2 - 4}$
143. $x + \sqrt{3 - 2x}$ $\frac{1}{2}x + \sqrt{5 - x}$
144. $2x + \sqrt{13 - 4x}$ $\frac{1}{2}x - \sqrt{x - 3}$
145. $2x + \sqrt{5 - x^2}$ $2x - \sqrt{x^2 - 12}$
146. $2x + \sqrt{x^2 - 4x - 23}$ $x + \sqrt{31 + 2x - x^2}$
147. $2x + \sqrt{61 - 16x + x^2}$ $2x - \sqrt{61 + 16x - x^2}$
148. $x - \sqrt{2x^2 + 10x + 13}$ $3x + \sqrt{24x - 54 - x^2}$

XXVII.

Quadratische Gleichungen mit zwei Unbekannten.

Für die Lösung quadratischer Gleichungen mit mehreren Unbekannten lassen sich nicht so allgemeine Regeln geben wie für die Lösung der Gleichungen vom ersten Grade mit mehreren Unbekannten.

Kommen in den Gleichungen nur x^2 und y^2 , oder ihre reziproken Werthe vor, so verfährt man wie bei den einfachen Gleichungen und bestimmt diese Größen zunächst. Dann hat man auch die Unbekannten selber (Nr. 1.—4.).

Ist die eine Gleichung eine einfache, in welcher also x und y nur in der ersten Potenz vorkommen, so führt die Substitutionsmethode

meistens am einfachsten zum Ziele (Nr. 5., 6. u. s. w.). Ganz so verfährt man, wenn man aus der Combination beider Gleichungen eine einfache ableiten kann.

Kann man aus einer Gleichung oder aus der Combination beider $\frac{x}{y}$ finden, so daß z. B. $\frac{x}{y} = \frac{m}{n}$ ist, so setzt man am einfachsten $x = mt$ und $y = nt$ und bestimmt zunächst t , womit dann auch x und y gefunden sind (Nr. 41. — 46.). Dies Verfahren findet auch Anwendung, wenn $\frac{x+y}{x-y}$ gegeben ist oder gefunden werden kann, da man dann auch $\frac{x}{y}$ kennt (Nr. 47., 48. u. s. w.).

In manchen Fällen ist es unbequem, die Unbekannten direkt zu suchen; man kommt einfacher und leichter zum Ziel, wenn man geeignete Ausdrücke von x und y als neue Unbekannte einführt und zunächst sucht. Solche Ausdrücke von x und y sind $x+y$, $x-y$, x^2+y^2 , xy , $\frac{x+y}{x-y}$, $\frac{x^2+y^2}{xy}$ u. s. w.

Ist $x+y = a$ gegeben oder gefunden, so setzt man am einfachsten $x-y = t$ und bestimmt zunächst t . Man hat $x = \frac{a+t}{2}$ und $y = \frac{a-t}{2}$ für x und y zu substituieren und die für t sich ergebende Gleichung nach t aufzulösen. — Ähnlich verfährt man, wenn $x-y$ gegeben ist.

Ist x^2+y^2 gegeben, so sucht man xy . Dann kann man leicht $x+y$ und $x-y$ finden, also auch x und y selbst. — Ist xy gegeben, so sucht man x^2+y^2 zunächst.

Kommt in einer Gleichung ein Ausdruck von x und y und sein reziproker Werth vor, so setzt man den Ausdruck gleich einer neuen Unbekannten u und bestimmt zunächst u . So läßt sich aus $a\frac{x}{y} + b\frac{y}{x} = c$ der Quotient $\frac{x}{y}$ finden. Ebenso läßt sich $\frac{x}{y}$ finden aus Gleichungen von der Form

$$ax^2 + bxy + cy^2 = 0$$

$$ax^2 + by^2 = cxy$$

$$\frac{mx^2 + nx + py^2}{m_1x^2 + n_1xy + p_1y^2} = \frac{a}{b}$$

Ist $\frac{x}{y}$ bestimmt, so hat die Auflösung keine Schwierigkeit mehr. Oft ist es freilich nöthig, um das Resultat in einer möglichst einfachen Form zu erhalten, den Quotienten $\frac{x}{y}$ zunächst auf eine geeignete Form zu bringen. Darüber sehe man A. G. S. 51 ff.

Wie man aus der Gleichung

$$\frac{mx^4 + nx^3y + px^2y^2 + nxy^3 + my^4}{m_1x^4 + n_1x^3y + p_1x^2y^2 + n_1xy^3 + m_1y^4} = \frac{a}{b}$$

den Quotienten $\frac{x}{y}$ findet und ihn in einer geeigneten Form darstellt, darüber sehe man N. G. S. 174 ff.

Auch die Zerlegung der Gleichungen leistet oft wesentliche Dienste. So zerfällt bei Nr. 39. die zweite Gleichung in 1) $x - 2 = 0$ und 2) $y - 1 = 0$. Beide Gleichungen sind der Reihe nach mit der ersten zu combiniren.

Erste Stufe.

- | | |
|--|---|
| 1. $\left \begin{array}{l} 5x^2 + 2y^2 = 22 \\ 3x^2 - 5y^2 = 7 \end{array} \right $ | 2. $\left \begin{array}{l} 2x^2 - 3y^2 = 6 \\ 3x^2 - 2y^2 = 19 \end{array} \right $ |
| 3. $\left \begin{array}{l} ax^2 - \frac{b}{y^2} = 2(a^2 - b^2) \\ bx^2 - \frac{a}{y^2} = a^2 - b^2 \end{array} \right $ | 4. $\left \begin{array}{l} \frac{a}{x^2} - \frac{b}{y^2} = a^2 + b^2 \\ \frac{b}{x^2} + \frac{a}{y^2} = a^2 + b^2 \end{array} \right $ |
| 5. $\left \begin{array}{l} x^2 + y^2 = 40 \\ x = 3y \end{array} \right $ | 6. $\left \begin{array}{l} 3x - y = 5 \\ xy - x = 0 \end{array} \right $ |
| 7. $\left \begin{array}{l} xy = 12 \\ 2x + 3y = 18 \end{array} \right $ | 8. $\left \begin{array}{l} x^2 + y^2 = 50 \\ 9x + 7y = 70 \end{array} \right $ |
| 9. $\left \begin{array}{l} 5x^2 + y = 3xy \\ 2x - y = 0 \end{array} \right $ | 10. $\left \begin{array}{l} x^2 - xy + y^2 = 7 \\ 2x - 3y = 0 \end{array} \right $ |
| 11. $\left \begin{array}{l} 3x^2 - 4y = 5x - 2y^2 \\ 3x + 4y = 10 \end{array} \right $ | 12. $\left \begin{array}{l} (x + y)(x - 2y) = 7 \\ x - y = 3 \end{array} \right $ |
| 13. $\left \begin{array}{l} (3x - 2y)(2x - 3y) = 26 \\ x - 2y + 1 = 0 \end{array} \right $ | |
| 14. $\left \begin{array}{l} x^2 + 2xy - y^2 = 7(x - y) \\ 2x - y = 5 \end{array} \right $ | |
| 15. $\left \begin{array}{l} 2x^2 - 5xy + y^2 + 10x + 12y = 100 \\ 2x - 3y = 1 \end{array} \right $ | |
| 16. $\left \begin{array}{l} 7(x + 5)^2 - 9(y + 4)^2 = 118 \\ x - y = 1 \end{array} \right $ | |
| 17. $\left xy = (3 - x)^2 = (2 - y)^2 \right $ | |
| 18. $\left xy = x^2 - y^2 = 2(x + y) \right $ | |
| 19. $\left \begin{array}{l} \frac{3x-2}{y+5} + \frac{y}{x} = 2 \\ x - y = 4 \end{array} \right $ | 20. $\left \begin{array}{l} \frac{2x-5}{x-2} + \frac{2y-3}{y-1} = 2 \\ 3x - 4y = 1 \end{array} \right $ |

$$21. \left| \begin{array}{l} \frac{2x-y+1}{x-2y+1} = \frac{8}{3} \\ x^2 - 3xy + y^2 = 5 \end{array} \right| \quad 22. \left| \begin{array}{l} \frac{2x-1}{x+1} + \frac{y+3}{y+1} = 3 \\ x^2 + y^2 = 2x + y \end{array} \right|$$

$$23. \left| \begin{array}{l} \frac{1+x+x^2}{1+y+y^2} = 3 \\ x + y = 6 \end{array} \right| \quad 24. \left| \begin{array}{l} \frac{x^2+y+1}{y^2+x+1} = \frac{3}{2} \\ x - y = 1 \end{array} \right|$$

$$25. \left| \begin{array}{l} \frac{4x+y+3}{2x+y} - \frac{y+7}{x+3} = 1 \\ x + y = 10 \end{array} \right|$$

$$26. \left| \begin{array}{l} \frac{4x+y-1}{2x+y-1} - \frac{4x+y-12}{2x+y-12} = 3\frac{2}{3} \\ 3x + y = 13 \end{array} \right|$$

$$27. \left| \begin{array}{l} x + xy = 35 \\ y + xy = 32 \end{array} \right| \quad 28. \left| \begin{array}{l} x(y-1) = 10 \\ y(x-1) = 12 \end{array} \right|$$

$$29. \left| \begin{array}{l} x^2 + y^2 + x + y = 18 \\ x^2 - y^2 + x - y = 6 \end{array} \right| \quad 30. \left| \begin{array}{l} 2x^2 - 5xy + 3x - 2y = 10 \\ 5xy - 2x^2 + 7x - 8y = 10 \end{array} \right|$$

$$31. \left| \begin{array}{l} 5x + y + 3 = 2xy \\ xy = 2x - y + 9 \end{array} \right| \quad 32. \left| \begin{array}{l} (x+y)(8-x) = 10 \\ (x+y)(5-y) = 20 \end{array} \right|$$

$$33. \left| \begin{array}{l} (x-1)(y+5) = 100 \\ (x-2)(y+6) = 99 \end{array} \right| \quad 34. \left| \begin{array}{l} (x-2)^2 + (y+2)^2 = 17 \\ (x-1)^2 + (y+1)^2 = 13 \end{array} \right|$$

$$35. \left| \begin{array}{l} (x-4)(y+5) = 0 \\ (x+2)(y-3) = 0 \end{array} \right| \quad 36. \left| \begin{array}{l} (x+4)(y-3) = 0 \\ (x+7)(y-7) = 0 \end{array} \right|$$

$$37. \left| \begin{array}{l} 3x^2 - 2y^2 = 6(x-y) \\ xy = 0 \end{array} \right| \quad 38. \left| \begin{array}{l} 4x^2 - 9y^2 = 0 \\ 4x^2 + y^2 = 8(x+y) \end{array} \right|$$

$$39. \left| \begin{array}{l} 2x^2 - 3xy + 5y - 5 = 0 \\ (x-2)(y-1) = 0 \end{array} \right|$$

$$40. \left| \begin{array}{l} x^2 - 5y^2 - 3x - y + 22 = 0 \\ (x-3)(y-2) = y^2 - 3y + 2 \end{array} \right|$$

$$41. \left| \begin{array}{l} xy + 72 = 6(2x+y) \\ \frac{x}{y} = \frac{2}{3} \end{array} \right| \quad 42. \left| \begin{array}{l} x(x-y) - 5y = 6 \\ \frac{x}{x-y} = 3\frac{1}{2} \end{array} \right|$$

43. $\left| \begin{array}{l} x : y = 9 : 4 \\ x : 12 = 12 : y \end{array} \right|$
44. $\left| \begin{array}{l} x^2 + y = y^2 + x - 18 \\ x : y = 2 : 3 \end{array} \right|$
45. $\left| \begin{array}{l} x^2 + y^2 = a^2 \\ \frac{x}{y} = \frac{m}{n} \end{array} \right|$
46. $\left| \begin{array}{l} xy = a \\ \frac{x}{y} = b \end{array} \right|$
47. $\left| \begin{array}{l} x^2 + y^2 = 130 \\ \frac{x+y}{x-y} = 8 \end{array} \right|$
48. $\left| \begin{array}{l} (3x-y)(3y-x) = 36 \\ \frac{x+y}{x-y} = \frac{5}{2} \end{array} \right|$
49. $\left| \begin{array}{l} 3(x+y)^2 - 2(x-y)^2 = 73(x-y) \\ (2x-y) : (4x-3y) = 2 : 3 \end{array} \right|$
50. $\left| \begin{array}{l} ax^2 + (a-b)xy - by^2 = c^2 \\ (x+y) : (x-y) = a : b \end{array} \right|$
51. $\left| \begin{array}{l} ax - by = cy \\ a^2x^2 - b^2y^2 = acxy + m^2 \end{array} \right|$
52. $\left| \begin{array}{l} x^2 : y^2 = a^2 : b^2 \\ a - x = b - y \end{array} \right|$
53. $\left| \begin{array}{l} \frac{a}{x^2} - \frac{b}{y^2} = 0 \\ \frac{1}{x} + \frac{1}{y} = \frac{1}{c} \end{array} \right|$
54. $\left| \begin{array}{l} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \\ \frac{bx+ay}{bx-ay} = \frac{m}{n} \end{array} \right|$
55. $\left| \begin{array}{l} \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \frac{a}{b} \\ xy = (a^2 - b^2)^2 \end{array} \right|$
56. $\left| \begin{array}{l} \frac{x\sqrt{x} + y\sqrt{y}}{x\sqrt{x} - y\sqrt{y}} = \frac{a}{b} \\ x^3 - c^3 = c^3 - y^3 \end{array} \right|$
57. $\left| \begin{array}{l} x^2 + xy = a \\ y^2 + xy = b \end{array} \right|$
58. $\left| \begin{array}{l} x^3 + xy^2 = a \\ y^3 + x^2y = b \end{array} \right|$
59. $\left| \begin{array}{l} x^2y = a \\ xy^2 = b \end{array} \right|$
60. $\left| \begin{array}{l} x^2y + xy^2 = a \\ x^2y - xy^2 = b \end{array} \right|$
61. $\left| \begin{array}{l} x = a(x^2 + y^2) \\ y = b(x^2 + y^2) \end{array} \right|$
62. $\left| \begin{array}{l} x(x^3 + y^3) = a \\ y(x^3 + y^3) = b \end{array} \right|$
63. $\left| \begin{array}{l} x\sqrt{x+y} = a \\ y\sqrt{x+y} = b \end{array} \right|$
64. $\left| \begin{array}{l} x\sqrt[3]{x^2+y^2} = a \\ y\sqrt[3]{x^2+y^2} = b \end{array} \right|$
65. $\left| \begin{array}{l} (x+y)(x^2+y^2) = a \\ (x-y)(x^2+y^2) = b \end{array} \right|$
66. $\left| \begin{array}{l} x+y = a(x^2+y^2) \\ x-y = b(x^2+y^2) \end{array} \right|$
67. $| a(x-y) = b(x+y) = xy |$
68. $| a(x-y) = b(x+y) = x^2 + y^2 |$

69. $\left| \begin{array}{l} x^3 + y^3 = (a + b)(x - y)^2 \\ x^2 - xy + y^2 = (a - b)(x - y) \end{array} \right|$
70. $\left| \begin{array}{l} (x - y)(x + y)^2 = a \\ (x + y)(x - y)^2 = b \end{array} \right|$ 71. $\left| \begin{array}{l} (x + y)(x^2 + 3y^2) = a \\ (x - y)(x^2 + 3y^2) = b \end{array} \right|$
72. $\left| \begin{array}{l} x^3 + x^2y + xy^2 + y^3 = \frac{a}{x + y} \\ x^3 - x^2y + xy^2 - y^3 = \frac{b}{x - y} \end{array} \right|$
73. $\left| \begin{array}{l} x^3 + x^2y + xy^2 + y^3 = a^2(x - y) \\ x^3 - x^2y + xy^2 - y^3 = b^2(x + y) \end{array} \right|$
74. $\left| \begin{array}{l} x^2(1 + y + y^2 + y^3) = a \\ x^2(1 - y + y^2 - y^3) = b \end{array} \right|$
75. $\left| \begin{array}{l} x^2 + y^2 = a \\ xy = b \end{array} \right|$ 76. $\left| \begin{array}{l} x^2 + y^2 = 130 \\ xy = 63 \end{array} \right|$
77. $\left| \begin{array}{l} x^2 - y^2 = 40 \\ xy = 21 \end{array} \right|$ 78. $\left| \begin{array}{l} x^n + y^n = 2a \\ xy = b \end{array} \right|$
79. $\left| \begin{array}{l} x + y = a \\ x^2 + y^2 = b \end{array} \right|$ 80. $\left| \begin{array}{l} x^2 + y^2 = 250 \\ x - y = 4 \end{array} \right|$
81. $\left| \begin{array}{l} x + y = a \\ xy = b \end{array} \right|$ 82. $\left| \begin{array}{l} x - y = 5 \\ xy = 36 \end{array} \right|$
83. $\left| \begin{array}{l} x + y = 58 \\ \sqrt{x} + \sqrt{y} = 10 \end{array} \right|$ 84. $\left| \begin{array}{l} x + y = a \\ x^2 + y^2 = mxy \end{array} \right|$
85. $\left| \begin{array}{l} x^2 + xy + y^2 = 2a \\ x^2 - xy + y^2 = 2b \end{array} \right|$ 86. $\left| \begin{array}{l} x^2 - xy + y^2 = 39 \\ 2x^2 - 3xy + 2y^2 = 43 \end{array} \right|$
87. $\left| \begin{array}{l} x^2 + y^2 - 5(x + y) = 8 \\ x^2 + y^2 - 3(x + y) = 28 \end{array} \right|$ 88. $\left| \begin{array}{l} 3xy - 2(x + y) = 28 \\ 2xy - 3(x + y) = 2 \end{array} \right|$
89. $\left| \begin{array}{l} x + xy + y = 5 \\ x^2 + xy + y^2 = 7 \end{array} \right|$ 90. $\left| \begin{array}{l} x + xy + y = 11 \\ x^2 + x^2y^2 + y^2 = 49 \end{array} \right|$
91. $\left| \begin{array}{l} (x + y)^2 - 4(x + y) = 45 \\ (x - y)^2 - 2(x - y) = 3 \end{array} \right|$
92. $\left| \begin{array}{l} (2x - y)^2 - 12(2x - y) = 189 \\ x^2 - 4xy + 4y^2 - 3x + 6y = 54 \end{array} \right|$

$$93. \begin{cases} x^2 - xy + y^2 = 13(x - y) \\ xy = 12 \end{cases} \quad 94. \begin{cases} x^2 + y^2 - 2(x - y) = 38 \\ xy + 3(x - y) = 25 \end{cases}$$

$$95. \begin{cases} x^2 + y^2 + x - y = 12 \\ 2xy = 3(x - y) \end{cases} \quad 96. \begin{cases} x^2 + y^2 - 12 = x + y \\ xy + 8 = 2(x + y) \end{cases}$$

$$97. \begin{cases} \frac{1}{x} + \frac{1}{y} = 5 \\ x - y = 0,3 \end{cases} \quad 98. \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{1}{3} \\ x^2 + y^2 = 160 \end{cases}$$

$$99. \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{3}{2} \\ \frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{4} \end{cases} \quad 100. \begin{cases} \frac{x^2}{m^2} + \frac{y^2}{n^2} = 1 \\ \frac{ax}{m} + \frac{by}{n} = 1 \end{cases}$$

$$101. \begin{cases} x \left(1 + \frac{x}{y}\right) = a \\ y \left(1 + \frac{y}{x}\right) = b \end{cases} \quad 102. \begin{cases} \frac{x+1}{y+1} = 2 \\ \frac{x^2+1}{y^2+1} = 5 \end{cases}$$

$$103. \begin{cases} x + y = a \\ x^3 + y^3 = b \end{cases} \quad 104. \begin{cases} \sqrt[3]{x} + \sqrt[3]{y} = a \\ x + y = b \end{cases}$$

$$105. \begin{cases} ax + by = c \\ a^3x^3 + b^3y^3 = abxy \end{cases} \quad 106. \begin{cases} x^3 + 3xy^2 = a \\ y^3 + 3x^2y = b \end{cases}$$

$$107. \begin{cases} x + y^2 = ax \\ x^2 + y = by \end{cases} \quad 108. \begin{cases} ax^2 + by^2 = cx^3 \\ cx^2 - dy^2 = ax \end{cases}$$

$$109. \begin{cases} x^2 + ay^2 = \frac{a+1}{a-1} \\ ax^2 + y^2 = (a^2 - 1)y \end{cases} \quad 110. \begin{cases} \frac{x^3 + y^3}{x - y} = \frac{a^3 + b^3}{a - b} \\ x + y = a + b \end{cases}$$

$$111. \begin{cases} \frac{x-1}{y-1} = \frac{a-1}{b-1} \\ \frac{x^3-1}{y^3-1} = \frac{a^3-1}{b^3-1} \end{cases} \quad 112. \begin{cases} x = 10 \cdot \frac{y-1}{y+1} \\ y = \frac{9}{2} \cdot \frac{x-1}{x+1} \end{cases}$$

$$113. \begin{cases} \frac{(1+x)(1+y)}{(1-x)(1-y)} = a \\ \frac{(1+x)(1-y)}{(1-x)(1+y)} = b \end{cases} \quad 114. \begin{cases} xy + \frac{x}{y} = a(x^2 + y^2) \\ xy - \frac{x}{y} = b(x^2 + y^2) \end{cases}$$

$$115. \begin{cases} (x^2 + y^2) \frac{x+y}{x-y} = a^2 \\ (x^2 + y^2) \frac{x-y}{x+y} = b^2 \end{cases} \quad 116. \begin{cases} \frac{x^2 - xy + y^2}{x - y} = a \\ \frac{x^2 + xy + y^2}{x + y} = b \end{cases}$$

$$117. \left| \begin{array}{l} x^2 - xy + y^2 = \sqrt[3]{a^2 - ab + b^2} \\ x^3 + y^3 = a + b \end{array} \right|$$

$$118. \left| \begin{array}{l} x + \sqrt[3]{x^2 y} = a \\ y + \sqrt[3]{x y^2} = b \end{array} \right| \quad 119. \left| \begin{array}{l} \sqrt{x-5} + \sqrt{y+2} = 5 \\ x + y = 16 \end{array} \right|$$

$$120. \left| \begin{array}{l} \sqrt{5-3x+x^2} + \sqrt{5-3y+y^2} = 6 \\ x + y = 3 \end{array} \right|$$

$$121. \left| \begin{array}{l} \sqrt{3-x+\frac{1}{4}x^2} + \sqrt{3-y+\frac{1}{4}y^2} = 3 \\ x + y = 4 \end{array} \right|$$

$$122. \left| \begin{array}{l} \sqrt{x(1-y)} + \sqrt{y(1-x)} = a \\ x + y = b \end{array} \right|$$

Zweite Stufe.

$$1. \left| \begin{array}{l} \frac{x}{y} + \frac{y}{x} = \frac{25}{12} \\ x^2 - y^2 = 28 \end{array} \right| \quad 2. \left| \begin{array}{l} \frac{x}{y} - \frac{y}{x} = \frac{16}{15} \\ 3x^2 + 5y^2 = 120 \end{array} \right|$$

$$3. \left| \begin{array}{l} 3x^2 - 8xy + 4y^2 = 0 \\ x^2 + y^2 + 13(x-y) = 0 \end{array} \right| \quad 4. \left| \begin{array}{l} 2x^2 - 3xy + y^2 = 3 \\ x^2 + 2xy - 3y^2 = 5 \end{array} \right|$$

$$5. \left| \begin{array}{l} (2x+3y)(x-y) = 58 \\ (3x-2y)(x+y) = 132 \end{array} \right| \quad 6. \left| \begin{array}{l} (5x+3y)(3x-5y) = 72 \\ (4x-y)(x+4y) = 77 \end{array} \right|$$

$$7. \left| \begin{array}{l} x^2 - xy + y^2 = 37 \\ x^2 - y^2 = 40 \end{array} \right| \quad 8. \left| \begin{array}{l} (x+y)^2 = 3x^2 - 2 \\ (x-y)^2 = 3y^2 - 11 \end{array} \right|$$

$$9. \left| \begin{array}{l} x^2 - 2xy + 3y^2 = 3(x-y) \\ 2x^2 + xy - y^2 = 9(x-y) \end{array} \right|$$

$$10. \left| \begin{array}{l} 2x^2 - 3xy = 9(x-2y) \\ x^2 - 3y^2 = 6(x-2y) \end{array} \right| \quad 11. \left| \begin{array}{l} (5x-7y)^2 = 49(x-y) \\ (3x-5y)^2 = 9(x-y) \end{array} \right|$$

$$12. \left| \begin{array}{l} (x+2y)(x+3y) = 3(x+y) \\ (2x+y)(3x+y) = 28(x+y) \end{array} \right|$$

$$13. \left| \begin{array}{l} (2x-3y)(3x+4y) = 39(x-2y) \\ (3x+2y)(4x-3y) = 99(x-2y) \end{array} \right|$$

14. $| x^2 + y^2 = xy = x + y |$

15. $| x^2 + y^2 = 10xy - 5(x + y) = 5(xy - 1) |$

16. $| x^3 + y^3 = 7xy = 28(x + y) |$

17. $| x^3 + y^3 = 4x^2 - \frac{3}{4}xy + 4y^2 = 13(x + y) |$

18. $| 19x^2 - 26xy + 19y^2 = 91$

$| 47x^2 - 26xy + 47y^2 = 91(x + y) |$

19. $| \frac{x^2 + xy + y^2}{x^2 - xy + y^2} = \frac{a}{b} |$

$| x^3 + y^3 = 2b |$

20. $| \frac{x^2 + xy + y^2}{x^2 - xy + y^2} = a$

$| 4x^4 + 7x^2y^2 + 4y^4 = 16 |$

21. $| (x^2 + y^2)(x + y) = a |$

$| xy(x + y) = b |$

22. $| x^3 + y^3 = a(x^2 + y^2) |$

$| x^2y + xy^2 = b(x^2 + y^2) |$

23. $| x^2 + xy + y^2 = \frac{a}{x^2 + y^2} = \frac{b}{xy} |$

24. $| (x + y)(x^2 + y^2) = \frac{1}{2}b \left(\frac{x}{y} + \frac{y}{x} \right) = a |$

25. $| x^4 + x^2y^2 + y^4 = a |$

$| x^2 + xy + y^2 = \frac{b}{xy} |$

26. $| x^4 - x^2y^2 + y^4 = 4a |$

$| x^2 - xy + y^2 = \frac{2b}{xy} |$

27. $| (x + y)(x^2 - y^2) = a |$

$| (x - y)(x^2 + y^2) = b |$

28. $| (x + y)(x^2 + y^2) = a |$

$| (x - y)(x^2 - y^2) = b |$

29. $| (x^2 + xy + y^2) \sqrt{x^2 + y^2} = a |$

$| (x^2 - xy + y^2) \sqrt{x^2 + y^2} = b |$

30. $| x^3 - y^3 = \frac{a}{x + y} |$

$| x^3 + y^3 = \frac{b}{x - y} |$

31. $| x^3 - y^3 = a \sqrt{\frac{x - y}{x + y}} |$

$| x^3 + y^3 = b \sqrt{\frac{x + y}{x - y}} |$

32. $| (a - x)^2 - 2(b - y)^2 = (a - x)(b - y) |$

$| x - y = 3(a - b) |$

33. $| (a - x)^2 + (b - y)^2 = c |$

$| (a - x)(b - y) = m |$

34. $| (7 + x)^2 + (5 - y)^2 = 109 |$

$| (7 + x)(5 - y) = 30 |$

35. $| \frac{a - x}{b - y} + \frac{b - y}{a - x} = \frac{34}{15} |$

$| x - y = 3(a - b) |$

36. $| \frac{x - 3}{8 - y} + \frac{y - 8}{x - 3} = \frac{16}{15} |$

$| x + y = 13 |$

37. $\left| \begin{array}{l} x + y = a \\ \frac{x}{b-y} + \frac{b-y}{x} = \frac{5}{2} \end{array} \right|$
38. $\left| \begin{array}{l} x - y = m \\ \frac{(a-x)^2 + y^2}{(a-x)y} = \frac{13}{6} \end{array} \right|$
39. $\left| \begin{array}{l} \frac{(a-x)^2 + (a-x)y + y^2}{(a-x)^2 - (a-x)y + y^2} = \frac{49}{19} \\ x - y = b \end{array} \right|$
40. $\left| \begin{array}{l} x^2 y = (a-x)^3 \\ x y^2 = (b-y)^3 \end{array} \right|$
41. $\left| \begin{array}{l} x^3 = (a-x)^2 (b-y) \\ y^3 = (a-x) (b-y)^2 \end{array} \right|$
42. $\left| \begin{array}{l} x + y = a \\ x^4 + y^4 = b \end{array} \right|$
43. $\left| \begin{array}{l} x + y = a \\ x^5 + y^5 = b \end{array} \right|$
44. $\left| \begin{array}{l} x + y = a \\ \frac{x^3}{y} + \frac{y^3}{x} = b \end{array} \right|$
45. $\left| \begin{array}{l} x + y = a \\ \frac{x^3}{y^2} + \frac{y^3}{x^2} = b \end{array} \right|$
46. $\left| \begin{array}{l} \frac{x^5 + y^5}{x^3 + y^3} = \frac{121}{13} \\ x + y = 2 \end{array} \right|$
47. $\left| \begin{array}{l} \frac{x^5 + y^5}{x^4 + y^4} = \frac{122}{41} \\ x + y = 4 \end{array} \right|$
48. $\left| \begin{array}{l} x + y = a \\ (m+x)^3 + (n+y)^3 = 72 \end{array} \right|$
49. $\left| \begin{array}{l} \sqrt[3]{m+x} + \sqrt[3]{n+y} = a \\ x + y = b \end{array} \right|$
50. $\left| \begin{array}{l} x - y = 3 \\ (x-4)^3 + (7-y)^3 = 72 \end{array} \right|$
51. $\left| \begin{array}{l} x + y = 444 \\ \sqrt[3]{x+10} + \sqrt[3]{y+14} = 12 \end{array} \right|$
52. $\left| \begin{array}{l} x + y = 3 \\ (x-1)^4 + (y+1)^4 = 17 \end{array} \right|$
53. $\left| \begin{array}{l} x - y = 1 \\ \sqrt[4]{6+x} + \sqrt[4]{10-y} = 3 \end{array} \right|$
54. $\left| \begin{array}{l} x + y = 2 \\ (x+1)^5 + (y-2)^5 = 211 \end{array} \right|$
55. $\left| \begin{array}{l} x - y = 50 \\ \sqrt[5]{143+x} - \sqrt[5]{y-18} = 1 \end{array} \right|$
56. $\left| \begin{array}{l} (x^2 - y + 1)(y^2 - x + 1) = 16 \\ x + y = 5 \end{array} \right|$
57. $\left| \begin{array}{l} (x^2 - x + 1)(y^2 - y + 1) = 3 \\ (x+1)(y+1) = 6 \end{array} \right|$
58. $\left| \begin{array}{l} x^2 + y = 5(x-y) \\ x + y^2 = 2(x-y) \end{array} \right|$
59. $\left| \begin{array}{l} x + y^2 = a(x^2 - y^2) \\ x^2 + y = b(x^2 - y^2) \end{array} \right|$
60. $| x^2 + y = x + y^2 = 4\frac{1}{2}xy |$
61. $| x^2 + y = x + y^2 = 1\frac{3}{8}(x^2 + y^2) |$

62. $\left| \begin{array}{l} x^3 + x^3y^3 + y^3 = 17 \\ x + xy + y = 5 \end{array} \right|$ 63. $\left| \begin{array}{l} x^2 - xy + y^2 = 2a \\ x^4 - x^2y^2 + y^4 = 2b \end{array} \right|$
64. $\left| \begin{array}{l} x^4 + x^2y^2 + y^4 = \frac{ax - by}{x^2 - y^2} \\ xy(x^2 + y^2) = \frac{ay - bx}{x^2 - y^2} \end{array} \right|$
65. $\left| \begin{array}{l} x(x+y)(x+2y)(x+3y) = 120 \\ 2x + 3y = 7 \end{array} \right|$
66. $\left| \begin{array}{l} x^4 + y^4 = 2a(x^2 + y^2) \\ xy = b \end{array} \right|$ 67. $\left| \begin{array}{l} x^2 + y^2 = a \\ x^4 + y^4 = 2bxy \end{array} \right|$
68. $\left| \begin{array}{l} x^2 + y^2 = a \\ \frac{x^5 + y^5}{x^3 + y^3} = b \end{array} \right|$ 69. $\left| \begin{array}{l} xy = 2a \\ \frac{x^5 + y^5}{x^3 + y^3} = 2b \end{array} \right|$
70. $\left| \begin{array}{l} x^2 + y^2 = a \\ \frac{x^3 + y^3}{x^3 - y^3} = b \cdot \frac{x-y}{x+y} \end{array} \right|$ 71. $\left| \begin{array}{l} x^2 + y^2 = m \\ \frac{x^3}{y^3} = \frac{bx - ay}{ax - by} \end{array} \right|$
72. $\left| \begin{array}{l} x^4 + y^4 = a(x+y)^2 \\ xy = b(x+y) \end{array} \right|$ 73. $\left| \begin{array}{l} x^4 + y^4 = 2a(x+y)^2 \\ x^2 + y^2 = 2b(x+y) \end{array} \right|$
74. $\left| \begin{array}{l} x^2 - y^2 = a \\ \frac{x+y}{x-y} = \frac{c}{x^2 + y^2} \end{array} \right|$ 75. $\left| \begin{array}{l} x^2 - y^2 = a \\ \frac{x^3 + y^3}{x-y} = c \end{array} \right|$
76. $\left| \begin{array}{l} \frac{x+x^2}{y+y^2} = 4\frac{2}{3} \\ \frac{y+x^2}{x+y^2} = 3\frac{1}{4} \end{array} \right|$ 77. $\left| \begin{array}{l} \frac{1+x+x^2}{1+y+y^2} = 4\frac{1}{3} \\ \frac{1+y+x^2}{1+x+y^2} = 3\frac{4}{13} \end{array} \right|$
78. $\left| \begin{array}{l} \frac{x+1}{y+1} = \frac{2}{3} \left(\frac{x-1}{y-1} \right) \\ \frac{x^2+x+1}{y^2+y+1} = \frac{13}{28} \left(\frac{x-1}{y-1} \right)^2 \end{array} \right|$
79. $\left| \begin{array}{l} \frac{x+1}{y+1} = \frac{3}{4} \left(\frac{x-1}{y-1} \right) \\ \frac{x^2+x+1}{y^2+y+1} = \frac{31}{39} \left(\frac{x^2-x+1}{y^2-y+1} \right) \end{array} \right|$
80. $\left| \begin{array}{l} \frac{(1+x)(1+y)}{(1-x)(1-y)} = 3 \\ \frac{(1+x^2)(1+y^2)}{(1-x^2)(1-y^2)} = \frac{65}{48} \end{array} \right|$

$$81. \left| \begin{array}{l} \frac{y(1+x^2)}{x(1+y^2)} = a \\ \frac{y(1-x^2)}{x(1-y^2)} = b \end{array} \right|$$

$$82. \left| \begin{array}{l} \frac{y(1+x^2)}{x(1+y^2)} = \frac{5}{3} \\ \frac{y^2(1+x^4)}{x^2(1+y^4)} = \frac{41}{9} \end{array} \right|$$

$$83. \left| \begin{array}{l} \frac{x+y}{x-y} = \frac{3}{2}xy \\ \frac{xy+1}{xy-1} = \frac{3x}{2y} \end{array} \right|$$

$$84. \left| \begin{array}{l} \sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} = \frac{x-y}{a} \\ \frac{x}{y} \cdot \frac{m^2+xy}{m^2-xy} = c^2 \end{array} \right|$$

$$85. \left| \begin{array}{l} \frac{x^2+y^2}{xy} = a \\ \frac{1+x^2y^2}{xy} = b \end{array} \right|$$

$$86. \left| \begin{array}{l} \frac{x+y}{1+xy} = a \\ \frac{x-y}{1-xy} = b \end{array} \right|$$

$$87. \left| \begin{array}{l} \frac{x+y}{1+xy} = \frac{2a}{1+a^2} \\ \frac{x-y}{1-xy} = \frac{2b}{1+b^2} \end{array} \right|$$

$$88. \left| \begin{array}{l} \frac{x+y}{1+xy} = \frac{a^2-m^2}{a^2+m^2} \\ \frac{x-y}{1-xy} = \frac{b^2-n^2}{b^2+n^2} \end{array} \right|$$

$$89. \left| \begin{array}{l} \frac{x+y}{1-xy} = \frac{2a}{1-a^2} \\ \frac{x-y}{1+xy} = \frac{2b}{1-b^2} \end{array} \right|$$

$$90. \left| \begin{array}{l} \frac{x+y}{1-xy} = 31 \\ \frac{x-y}{1+xy} = \frac{11}{29} \end{array} \right|$$

$$91. \left| \begin{array}{l} \frac{x+y}{1+xy} = \frac{1}{2} \\ \frac{x^2+xy+y^2}{1+xy+x^2y^2} = \frac{49}{241} \end{array} \right|$$

$$92. \left| \begin{array}{l} \sqrt{xy} + \sqrt{(1-x)(1-y)} = a \\ \sqrt{x(1-y)} + \sqrt{y(1-x)} = b \end{array} \right|$$

$$93. \left| \begin{array}{l} xy + \sqrt{(1-x^2)(1-y^2)} = a \\ x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sqrt{1-b^2} \end{array} \right|$$

$$94. \left| \begin{array}{l} \sqrt{x(1-y)} + \sqrt{y(1-x)} = a \\ \sqrt{x(1-x)} + \sqrt{y(1-y)} = b \end{array} \right|$$

Dritte Stufe.

$$1. \left| \begin{array}{l} \frac{x^3}{y} + \frac{y^3}{x} = a^2 - 2ab - b^2 \\ x^2 + xy + y^2 = ab \end{array} \right|$$

2. $\left| \begin{array}{l} x^4 + y^4 = (2a^2 - b^2)(x^2 + y^2) \\ x^2 + xy + y^2 = a(2a - b) \end{array} \right|$
3. $\left| \begin{array}{l} \frac{x^3 + y^3}{a^3 + b^3} = \frac{a - b}{x - y} = \frac{x + y}{a + b} \end{array} \right|$
4. $\left| \begin{array}{l} \frac{1}{x} + \frac{1}{y} = \frac{1}{a} + \frac{1}{b} \\ x^2 + y^2 = a^2 + b^2 \end{array} \right|$
5. $\left| \begin{array}{l} \frac{1}{x^3} + \frac{1}{y^3} = \frac{1}{a^3} + \frac{1}{b^3} \\ x^2y + xy^2 = a^2b + ab^2 \end{array} \right|$
6. $\left| \begin{array}{l} x + y = a + b \\ \frac{x^3 + y^3}{a^3 + b^3} = \frac{x - y}{a - b} \end{array} \right|$
7. $\left| \begin{array}{l} x + y = a + b \\ \frac{x^4 + y^4}{a^4 + b^4} = \left(\frac{x - y}{a - b}\right)^2 \end{array} \right|$
8. $\left| \begin{array}{l} x^2 - xy + y^2 = a^2 - ab + b^2 \\ \frac{x^5 + y^5}{a^5 + b^5} = \frac{x^3 + y^3}{a^3 + b^3} \end{array} \right|$
9. $\left| \begin{array}{l} \frac{x^3 - y^3}{x^2 - y^2} = \frac{a^3 - b^3}{a^2 - b^2} \\ \frac{x^5 - y^5}{x^4 - y^4} = \frac{a^5 - b^5}{a^4 - b^4} \end{array} \right|$
9. $\left| \begin{array}{l} \frac{x^3 + y^3}{x^2 - y^2} = \frac{a^3 + b^3}{a^2 - b^2} \\ \frac{x^4 + y^4}{x^3 - y^3} = \frac{a^4 + b^4}{a^3 - b^3} \end{array} \right|$
10. $\left| \begin{array}{l} (x + y)^3 = a(x^2 + y^2) \\ xy = b(x + y) \end{array} \right|$
11. $\left| \begin{array}{l} x^2 - y^2 = a \\ 4xy(x^2 + y^2) = b \end{array} \right|$
12. $\left| \begin{array}{l} xy(x + y) = a \\ x^5 + y^5 = bxy \end{array} \right|$
13. $\left| \begin{array}{l} xy(x + y) = a \\ x^5y^5(x^5 + y^5) = b \end{array} \right|$
14. $\left| \begin{array}{l} x^4 + x^3y + x^2y^2 + xy^3 + y^4 = a \\ x^4 - x^3y + x^2y^2 - xy^3 + y^4 = a \end{array} \right|$
15. $\left| \begin{array}{l} x^4 + x^3y + x^2y^2 + xy^3 + y^4 = a \\ x^8 + x^6y^2 + x^4y^4 + x^2y^6 + y^8 = b \end{array} \right|$
16. $\left| \begin{array}{l} (x + y)^4 = a(x^2 + y^2) \\ x^4 + y^4 = b(x^2 + y^2) \end{array} \right|$
17. $\left| \begin{array}{l} (x + y)^4 = 2axy \\ x^4 + y^4 = 2bxy \end{array} \right|$
18. $\left| \begin{array}{l} x^3 + y^3 = axy \\ x^5 + y^5 = bx^2y^2 \end{array} \right|$
19. $\left| \begin{array}{l} x^4 + y^4 = a(x + y)^2 \\ x^5 + y^5 = b(x + y)^3 \end{array} \right|$
20. $\left| \begin{array}{l} \frac{x^2 + y^2}{x + y} = a \\ \frac{x^4 + y^4}{x^3 + y^3} = b \end{array} \right|$
21. $\left| \begin{array}{l} \frac{x^3 + y^3}{x^2 + y^2} = a \\ \frac{x^5 - y^5}{x^4 - y^4} = b \end{array} \right|$
22. $\left| \begin{array}{l} (x^2 + y^2)(x^3 + y^3) = a \\ (x + y)(x^4 + y^4) = b \end{array} \right|$
23. $\left| \begin{array}{l} (x^2 + y^2)(x^3 + y^3) = axy \\ (x + y)(x^4 + y^4) = bxy \end{array} \right|$

24. $\left| \begin{array}{l} (x+y)^5 = 10a(x^3+y^3) \\ x^5+y^5 = 10b(x^3+y^3) \end{array} \right|$ 25. $\left| \begin{array}{l} x^5+y^5 = a(x^3+y^3) \\ x^2y^2(x+y) = b(x^3+y^3) \end{array} \right|$
26. $\left| \begin{array}{l} (x-y)(x^2-y^2)(x^3-y^3)(x^4-y^4) = a \\ (x+y)(x^2+y^2)(x^2+xy+y^2) = b \end{array} \right|$
27. $\left| \begin{array}{l} (x-y)^2(x^2-y^2)(x^4-y^4) = a \\ (x+y)^2(x^2+y^2)(x^4+y^4) = b \end{array} \right|$
28. $\left| \begin{array}{l} (x+y)(x^3+y^3) = axy \\ (x-y)(x^3-y^3) = bxy \end{array} \right|$ 29. $\left| \begin{array}{l} (x+y)(x^3+y^3) = 4a \\ (x-y)(x^3-y^3) = 4b \end{array} \right|$
30. $\left| \begin{array}{l} (x+y)^2(x^2+xy+y^2) = 3a(x^2+y^2) \\ (x-y)^2(x^2-xy+y^2) = 3b(x^2+y^2) \end{array} \right|$
31. $\left| \begin{array}{l} (x^2+xy+y^2)(x+y)^2 = a(5x^2+7xy+5y^2) \\ x^4+y^4 = b(5x^2+7xy+5y^2) \end{array} \right|$
32. $|axy(x^2+y^2) = ab(x^3+y^3) = b(x^4+y^4)|$
33. $|a(x^5+y^5) = ab(x^4+y^4) = bxy(x^3+y^3)|$
34. $|a(x^5+y^5) = ab(x+y) = bxy(x^3+y^3)|$
35. $\left| \begin{array}{l} (x+y)^2(x^2+xy+y^2) = 12a \\ (x-y)^2(x^2-xy+y^2) = 12b \end{array} \right|$
36. $\left| \begin{array}{l} x^4 = ax + by \\ y^4 = bx + ay \end{array} \right|$ 37. $\left| \begin{array}{l} x^5 = 2ax - by \\ y^5 = 2ay - bx \end{array} \right|$
38. $\left| \begin{array}{l} (x+y)(x^4+y^4) = a \\ (x-y)(x^4-y^4) = b \end{array} \right|$ 39. $\left| \begin{array}{l} (x+y)(x^5-y^5) = a \\ (x-y)(x^5+y^5) = b \end{array} \right|$
40. $\left| \begin{array}{l} x^2y - xy^2 = \frac{1}{a}(x^4+y^4) \\ x^2y + xy^2 = b(x^2-y^2) \end{array} \right|$ 41. $\left| \begin{array}{l} x^2y - xy^2 = \frac{1}{a}(x^5+y^5) \\ x^2y + xy^2 = b(x-y) \end{array} \right|$
42. $\left| \begin{array}{l} x^4 + x^2y^2 + y^4 = a(x-y)^2 \\ x^4 + y^4 = b(x-y)^2 \end{array} \right|$
43. $\left| \begin{array}{l} x^2 + y^2 = a(x+y) \\ \frac{x^3+y^3}{x^3-y^3} = \frac{1}{b}(x-y) \end{array} \right|$ 44. $\left| \begin{array}{l} x^2 + y^2 = a(x+y) \\ \frac{x^4+y^4}{x^4-y^4} = \frac{1}{b}(x-y) \end{array} \right|$
45. $\left| \begin{array}{l} \frac{x^2-xy+y^2}{x+y} = a \\ \frac{x^2+xy+y^2}{x+y} = \frac{(x-y)^2}{b} \end{array} \right|$

$$46. \left| \begin{array}{l} x^4 - y^4 = a(x^3 - y^3) \\ x^3 + y^3 = b(x^2 + y^2) \end{array} \right| \quad 47. \left| \begin{array}{l} x^3 + y^3 = a(x^2 - y^2) \\ x^4 + y^4 = b(x^3 - y^3) \end{array} \right|$$

$$48. \left| \begin{array}{l} x^4 + x^2 y^2 + y^4 = 2a(x + y)^2 \\ x^5 + y^5 = 2b(x + y)^3 \end{array} \right|$$

$$49. \left| \begin{array}{l} x(x + y)(x + 2y)(x + 3y) = 840 \\ (x + y)^2 - (x - y)^2 = 11 \end{array} \right|$$

$$50. \left| \begin{array}{l} \frac{1 + xy}{x + y} + \frac{x + y}{1 + xy} = 2a \\ \frac{1 - xy}{x - y} + \frac{x - y}{1 - xy} = 2b \end{array} \right|$$

$$51. \left| \begin{array}{l} \frac{1 - xy}{x + y} + \frac{x + y}{1 - xy} = \frac{2}{a} \\ \frac{1 + xy}{x - y} + \frac{x - y}{1 + xy} = \frac{2}{b} \end{array} \right|$$

XXVIII.

Quadratische Gleichungen mit drei und vier Unbekannten.

A. Mit drei Unbekannten.

$$1. \left| \begin{array}{l} x : y : z = a : b : c \\ x^2 + y^2 + z^2 = m^2 \end{array} \right|$$

$$2. \left| \begin{array}{l} x(x + y + z) = a \\ y(x + y + z) = b \\ z(x + y + z) = c \end{array} \right|$$

$$3. \left| \begin{array}{l} (y + z)(x + y + z) = a \\ (x + z)(x + y + z) = b \\ (x + y)(x + y + z) = c \end{array} \right|$$

$$4. \left| \begin{array}{l} (y + z) : (x + z) : (x + y) = a : b : c \\ (y + z)^2 + (x + z)^2 + (x + y)^2 = 1 \end{array} \right|$$

$$5. \left| \begin{array}{l} 2x - 4y + z = 0 \\ x + y - 4z = 0 \\ (x + 1)(z + 1) = (y - 1)(y + 6) \end{array} \right|$$